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Simple model for response of bridges to non-uniform seismic excitation

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ABSTRACT

A simple engineering model is developed for evaluating the effects of non-uniform ("differential") seismic excitation on the response of long bridge-type structures. The model consists of two identical elastic piers connected through a rigid deck, subjected to different ground excitations at each support. The response of the system is obtained by decomposing the differential excitation into: (1) an anti-symmetric uniform excitation equal to the average of the two ground motions, and (2) a symmetric static load equal to the corresponding semi-difference. In the case of harmonic ground excitation in the form of a traveling SH wave, closed-form solutions are obtained for the response of the deck and the relative displacements (drift) of the piers. To compare the effects of differential excitation against a uniform excitation, two pertinent response ratios are introduced. It is shown that when the amplitude of the ground motion varies from point to point, the "equivalent" uniform excitation cannot be defined in a unique basis, and, accordingly, it is difficult to determine whether differential excitation increases or decrease the response of the structure. Results are presented in dimensioneless graphs and critical issues of the problem are discussed.

INTRODUCTION

Spatial variability of strong earthquake motion over small distances is a well-documented phenomenon. The effects of such non-uniform excitations on the response of long structures such as bridges and pipelines has been the subject of extensive research over the past 20 years. Most of these studies utilize probabilistic descriptions of the earthquake field aimed at predicting simultaneously both the extend and the consequences of spatial variability on structural response (Bolt 1987, Zerva et al 1988, Loh et al 1989, Harichandran and Wang 1990, Abrahamson et al 1991, Der Kiureghian and Neuenhofer 1992, Kasai et al 1997). On the other hand, less effort has been devoted in developing deterministic formulations of the problem, particularly single-degree-of-freedom structural models combined with wave-propagation -based descriptions of the spatially-varying seismic field (Scanlan 1976, Loh et al 1982, Mylonakis and Nikolaou 1993). The use of such models, although may not capture all the complexities of the real-life problem, could provide valuable insight to the physics of structural response which is often obscured by the complexity of more sophisticated formulations.

The scope of this paper is to present a simple deterministic model for predicting the response of a long bridge-type structure to non-uniform seismic excitation. The main features incorporated in the model are: (1) The structure consists of a rigid deck supported by two elastic piers of equal stiffness. (2) The soil under the structure is homogeneous and linearly elastic. (3) The foundations of the two piers follow exactly the motion imposed by the incoming seismic waves i.e., soil-structure interaction is neglected. Due to space limitations only steady-state response to harmonic excitation is presented herein. More information on the model can be found in Mylonakis and Nikolaou (1993) and in a forthcoming paper by Mylonakis et al (1999).

A FUNDAMENTAL DIFFERENTIALLY-EXCITED OSCILLATOR

The structural model studied in this paper is shown in Fig 1. It consists of a single-span frame supported by two piers of equal stiffness, K, connected through a rigid deck of length L and mass m. The structure is subjected to two time histories, $U_{g1}(t)$ and $U_{g2}(t)$, imposed at the first and second support, respectively. Having just one degree of freedom and two supports, the model represents the simplest possible structure subjected to differential ground excitation; it will thereafter be referred to as *Fundamental Differentially-Excited Oscillator*.

Decomposition of non-uniform seismic excitation into a symmetric and an anti-symmetric part

The problem of Fig 1 can be analyzed using general procedures for differentially-excited systems found in standard textbooks and commercial computer codes (Chopra 1995, Wilson 1997). The goal of this paper, however, is to explore

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the fundamental physics of the problem. Accordingly, alternative formulations are implemented aimed at maximizing the insight on effects of differential excitations. To this end, it is instructive to decompose the non-uniform ground excitation in two parts (Fig 1): an *anti-symmetric* part, $u_g^{(a)}$, equal to the average (in the time domain) of the two input motions, and a *symmetric* part, $u_g^{(s)}$, equal to the corresponding semi-difference. This is written as:

$$u_g^{(a)}(t) = \tfrac{1}{2}\left[u_{g1}(t) + u_{g2}(t)\right] \quad , \quad u_g^{(s)}(t) = \tfrac{1}{2}\left[u_{g1}(t) - u_{g2}(t)\right] \tag{1}$$

where the superscripts (s) and (a) stand for "symmetric" and "anti-symmetric", respectively.

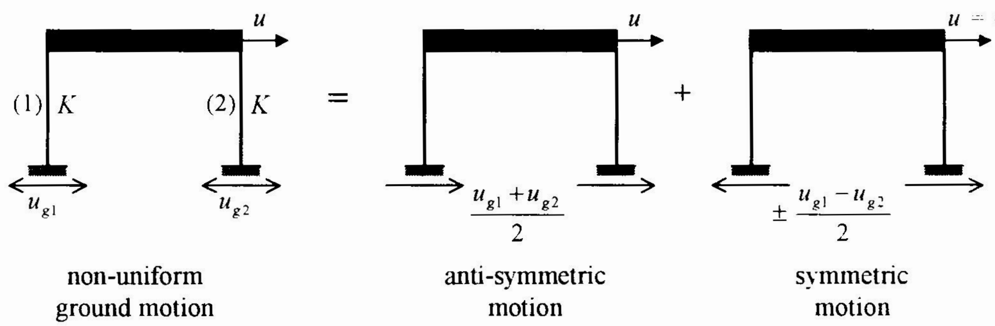

Figure 1. The model studied in this paper (left), and the decomposition of the excitation into a anti-symmetric and a symmetric and component (right). Note that the symmetric component does not excite the deck.

Based on Eqn (1), the problem of differential ground excitation is decomposed into two sub-problems: (1) a *dynamic* problem in which the structure undergoes a uniform ground motion corresponding to the anti-symmetric component $u_g^{(a)}$, and (2) a *static* problem in which the structure is subjected to the symmetric component $u_g^{(s)}$. Indeed, in the latter case the symmetry of the problem does not allow the deck to displace horizontally, making the excitation equivalent to a time-dependent static load (i.e., a "support movement"). It should be noted that the above decomposition is different than the traditional partition of structural response into a "rigid-body" motion and a "relative" (with respect to the ground) component. Similar, though somewhat more complicated, such models have been presented, among others, by Loh et al (1982) and Mylonakis et al (1999).

Response to harmonic steady-state excitation

In the case of harmonic ground excitation, the two supports are considered vibrating sinusoidally with the same cyclic frequency ω, but different amplitude and phase. Without loss of generality this can be expressed as:

$$u_{g1}(t) = d \sin \omega t \quad , \quad u_{g2}(t) = \eta \, d \sin(\omega t - \phi) \tag{2}$$

where d is the (maximum) amplitude of ground displacement at support 1 and η is a dimensionless factor ($0 \le \eta \le 1$); ϕ denotes the phase difference between the two motions. It is noted that trigonometric notation (instead of the perhaps more compact complex notation) is adopted throughout the paper to make the results more easily interpretable by engineers.

Based on Eqn (1), the anti-symmetric and symmetric components of the excitation are computed, respectively, as:

$$u_g^{(a)}(t) = \eta_a \, d \sin(\omega t - \phi_a) \quad , \quad u_g^{(s)}(t) = \eta_s \, d \sin(\omega t - \phi_s) \tag{3a}$$

where the new amplitude and phase coefficients are given by:

$$\eta_a = \tfrac{1}{2}\sqrt{\eta^2 + 2\eta\cos\phi + 1} \quad , \quad \phi_a = \arctan\left(\frac{\eta \sin\phi}{1+\eta\cos\phi}\right) \tag{3b}$$

$$\eta_s = \tfrac{1}{2}\sqrt{\eta^2 - 2\eta\cos\phi + 1} \quad , \quad \phi_s = \arctan\left(\frac{1-\eta\cos\phi}{\eta\sin\phi}\right) - \frac{\pi}{2} \tag{3c}$$

It can be observed that both η_a and η_s are smaller than 1 that is, the amplitudes of the symmetric and antisymmetric motions are *smaller* than the maximum displacement amplitude imposed to the structure.

Due to the symmetry of the system (Fig 1), the absolute motion of the deck is controlled exclusively by the antisymmetric component $u_g^{(a)}$. Accordingly, considering only steady-state vibrations, the motion of the deck, u(t), is obtained as

$$u(t) = \eta_a \, d \, R \sin(\omega t - \phi_a - \theta) \tag{4a}$$

where R and θ are the well-known *dynamic response factor* and *dynamic phase lag*, respectively (Chopra 1995)

$$R = \frac{\beta^2}{\sqrt{(1-\beta^2)^2 + (2\zeta\beta)^2}} \quad , \quad \theta = \tan^{-1}\left(\frac{2\zeta\beta}{1-\beta^2}\right) \tag{4b}$$

in which β $(= \omega/\omega_n)$ denotes the ratio between the excitation frequency ω and the natural frequency of the system ω_n $(= \sqrt{m/2K})$; ζ is the equivalent viscous damping ratio of the structure.

Special Case: Response to oblique SH waves

In the case of an SH wave traveling at an oblique angle trough the soil, non-uniform excitation is generated due to the difference in the arrival time, Δt, of the wave at the two supports. This phenomenon is known as the *wave passage effect* (Der Kiureghian and Neuenhofer 1992). Neglecting wave attenuation effects due to geometric and material damping in the soil, we may assume that the amplitude of ground motion is the same at the two supports (η = 1). Accordingly, the two components of the excitation (Eqns 3b and 3c) are simplified as

$$\eta_a = \cos\frac{\phi}{2} \quad , \quad \phi_a = \frac{\phi}{2} \tag{5a}$$

$$\eta_s = \sin\frac{\phi}{2} \quad , \quad \phi_s = \frac{\phi + \pi}{2} \tag{5c}$$

where $\phi = \omega \, \Delta t$. Note that for $\phi = 0$ the symmetric component of the motion vanishes (i.e., $\eta_s = 0$) and the problem reduces to a conventional uniform excitation. Similarly, for $\phi = \pi$ the motion in the two supports becomes out of phase; the anti-symmetric component vanishes ($\eta_a = 0$) and the seismic excitation becomes a time-dependent static load.

Dynamic Response Ratios

A question often raised in differential excitation analyses is whether the spatial variation increases or decreases the response of the structure. To answer this, the response of the structure is compared against a response caused by an "equivalent" uniform excitation (Loh et al 1982). As will be shown in later in this paper, this is not easy to do. The most important difficulty is that, if the non-uniform excitation varies in *amplitude* from point to point (i.e., if the earthquake response spectra are different at the various supports), the "equivalent" uniform excitation cannot be defined in a unique basis. In addition, different response parameters (accelerations, bending moments, axial forces) appear to be affected to different degrees spatially the excitation. In the particular case of a traveling wave, however, the amplitude of the motion is the same at the two supports. Accordingly, the equivalent uniform excitation can be uniquely determined by simply setting the phase difference angle ϕ in Eqn (2) equal to zero; thus

$$u_g^{(u)}(t) = d \sin \omega t \tag{6}$$

where the superscript (u) stands for "uniform". The steady-state response to this input is

$$u^{(u)}(t) = d \, R \sin(\omega t - \theta) \tag{7}$$

Comparing Eqns (4a) and (7) it is apparent that the ratio between the amplitudes of the two responses is equal to η_a. Taking the pertinent value for a traveling wave (Eqn 5) the following ratio is defined:

$$\Phi_s \equiv \left|\cos\frac{\phi}{2}\right| \leqslant 1 \tag{8}$$

which indicates that the wave passage effect *reduce*s the response of the deck. This result is reminiscent of corresponding results for the seismic response of rigid footings which is governed by wave interference effects (Scanlan 1976). Similarly, Eqn (8) can be interpreted as the result of destructive interference of seismic waves reaching the rigid deck through the two supports. It is interesting that Φ_s is independent of the vibrational characteristics of the structure (i.e., β and ζ). It is also noted that Eqn (8) applies to all absolute components of the deck motion (acceleration, velocity, displacement) as well as to the overall base shear. Accordingly Φ_s may be referred to as *Base Shear Ratio*. A set of such ratios has been proposed in a earlier study by Loh et al (1982). A graphical representation of Φ_s is shown in Figure 2.

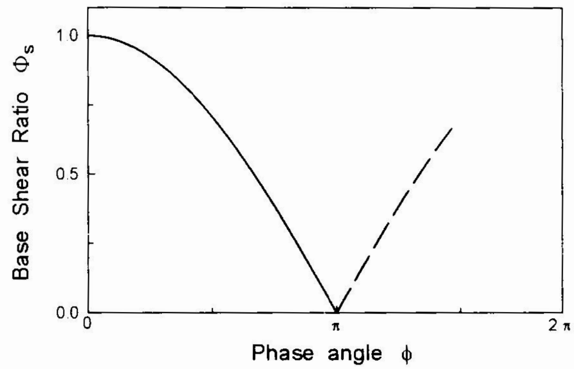

Figure 2. Ratio base shears in the structure caused by a harmonic ground motion with phase difference ϕ between the two supports, and a corresponding uniform motion. Note the independence of Φ_s to the vibrational characteristics (ω_n , ζ) of the structure.

Superimposing the responses due to the symmetric and anti-symmetric components of the excitation, the top-to-bottom relative displacements at columns 1 and 2 is obtained as

$$u_c(t)_{1,2} = u(t) \pm u^{(s)}(t) \tag{9}$$

in which the plus sign corresponds to support 1 and the minus to support 2 (Fig 1). Upon substituting Eqns (3a), (4a), and (5a), (5c) into Eqn (9) the maximum relative displacement, SD, in the columns is determined

$$SD_{1,2} = d\sqrt{\left(R\cos\frac{\phi}{2}\right)^2 + \left(\sin\frac{\phi}{2}\right)^2 \pm R\sin\phi\sin\theta} \tag{10}$$

The first term under the square root in Eqn (10) stands for the response due to the anti-symmetric (dynamic) component of the motion while the second is due to the symmetric (static) component. The third term corresponds to the coupling between the dynamic and the static response --- a "cross-correlation" term (Der Kiureghian and Neuenhofer 1992).

Taking the absolute value of the third term of Eqn (10), Eqn (10) takes its maximum value which corresponds to the *envelope* of relative displacements in the structure:

$$SD = d\sqrt{\left(R\cos\frac{\phi}{2}\right)^2 + \left(\sin\frac{\phi}{2}\right)^2 + R\,|\sin\phi|\sin\theta} \tag{11}$$

in which the superscripts 1 and 2 have been dropped to emphasize that result of (11) does not correspond to the response of a particular pier but merely to the overall maximum relative displacement. Equation 11 is illustrated in Fig 3 , plotted as function of the frequency ratio $\beta = \omega/\omega_n$ for different values of the angle ϕ . The following are noteworthy on this equation: *First*, the maximum column drift is always *larger* than the SRSS combination of the static and dynamic components. In fact, for resonant conditions, $\theta = \pi/2$ and $R = 1/2\zeta$, Eqn (11) simplifies to

$$SD = d\left(\frac{1}{2\zeta}\left|\cos\frac{\phi}{2}\right| + \left|\sin\frac{\phi}{2}\right|\right) \quad , \quad \omega = \omega_n \tag{12}$$

which implies that the drift is obtained by straight addition of the static and dynamic responses. However, the overall drift at resonance is always smaller to that obtained for a uniform excitation (ϕ = 0 --- see Fig 3). *Second*, for low-frequency excitations (β less than about 0.7) the drift becomes larger than that for ϕ = 0. Accordingly, neglecting differential effects in that frequency range may lead to unconservative estimates of seismic demand in the piers. This is in contrast to absolute deck accelerations which are always reduced by differential effects (Eqn 8). In the limiting case of $\beta \to 0$ (a stiff structure subjected to a long-period wave) the dynamic component vanishes altogether; thus

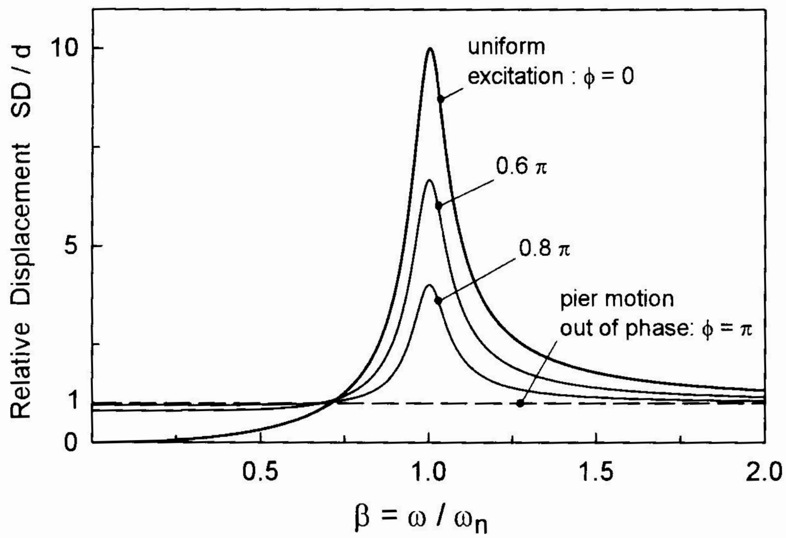

Figure 3. Envelope of top-to-bottom relative displacements in the piers due to a harmonic input with phase difference ϕ between the two supports; $\zeta = 5\%$

$$SD = d\left|\sin\frac{\phi}{2}\right| \quad , \quad \beta \to 0 \tag{13}$$

Such a finite relative displacement in a very-stiff structure does not develop in uniform excitations. *Third*, in the high-frequency excitation region ($\beta \to \infty$) the deck remains immovable in space; the maximum relative displacement in both columns becomes equal to d regardless of spatial variation. Indeed, taking the pertinent limit in Eqn (10) yields

$$SD_{1,2} \to d \quad , \quad \beta \to \infty \tag{14}$$

Relative displacements can be normalized in a similar manner as deck accelerations, by comparing with the relative displacements due of a uniform excitation. Indeed dividing Eqn (11) by SD = R d yield the ratio

$$\Phi_d = \sqrt{\cos^2\left(\frac{\phi}{2}\right) + R^{-2}\sin^2\left(\frac{\phi}{2}\right) + R^{-1}\left|\sin\phi\right|\,\sin\theta} \tag{15}$$

Clearly, Φ_d is applicable to both relative displacements and column drifts (i.e., relative displacements normalized by pier height). Accordingly, Φ_d is referred to as *Drift Ratio* (hence the subscript d). The ratio is illustrated in Fig 4, plotted for five different ϕ angles and 5% damping. For β larger than about 0.7 and ϕ different than zero, Φ_d is always smaller than 1 which implies that during high-frequency excitation differential effects tend to reduce column drift. In the high-frequency region, $\beta \to \infty$, Φ_d tends to unity regardless of ϕ . This is an expected result since in such high-frequencies the girder is practically motionless and accordingly relative displacements in each pier are equal to the corresponding ground displacement regardless of spatial variation. On the other hand, for low excitation frequencies ($\beta < 0.7$), Φ_d is larger than 1 and tends to infinity as β decreases. This trend, however, is rather misleading since the actual displacement demand in the piers is always finite (see Fig 3).

CONCLUSIONS

A simple model was presented for evaluating the effects of spatial variability in ground motion on the response of long bridge-type structures. The results reported in the paper are basically restricted to harmonic excitations with phase lag between the two structural supports, an effect analogous to that caused by a traveling seismic wave. The following conclusions were drawn from the study: (1) Phase lag in the motion of the supports reduces the absolute motion of the deck as compared to a corresponding uniform ground input. The reduction is independent of the vibrational characteristics of the structure. (2) The phase lag may increase or decrease the column drift depending on the value of the frequency ratio β. In the dynamic response range (β > 0.7), the drift decreases with increasing φ angles, while in the low-frequency range (β < 0.7) the trend reverses. Accordingly, differential effects may be of importance for stiff statically-indeterminate structures. (3) When the amplitude of the motion is different in the two supports, the

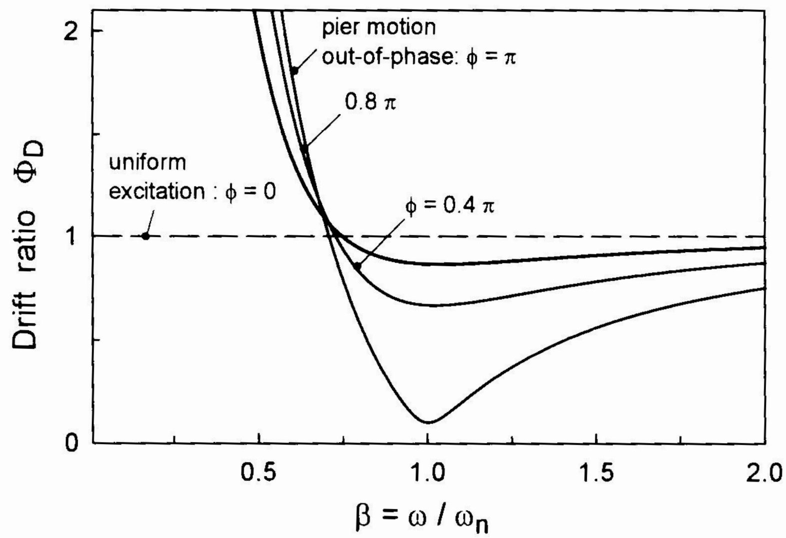

Figure 4. Ratio of maximum drift in the piers caused by a harmonic ground motion with phase difference φ between the two supports and a corresponding uniform motion; ζ = 5%.

"equivalent" uniform excitation cannot be defined in a unique basis. Accordingly, whether differential input increases or decreases the ground motion is not straightforward to determine.

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